

BOUNDS ON DISTRIBUTED TDOA-BASED LOCALIZATION OF OFDM SOURCES

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ABSTRACT

One main drawback of using Time Difference of Arrival (TDOA) methods for source localization and navigation is that they require centralization of multiple copies of a signal. This paper considers blindly estimating the location of a Cyclic Prefix (CP) in an Orthogonal Frequency Division Multiplexing (OFDM) signal, enabling distributed TDOA computation up to an integer ambiguity. This ambiguity can be resolved using integer least-squares methods, if enough TDOAs are available. The contributions of this paper are derivation of the Cramer-Rao Lower Bound (CRLB) on locating the CP, and hence on the underlying source localization problem.

Index Terms— Cramer-Rao lower bound, OFDM, cyclic prefix, time delay estimation, blind, TDOA

1. INTRODUCTION

One main drawback of using TDOA methods for source localization and navigation is that they require centralization of multiple copies of a signal, which wastes bandwidth and power. For OFDM sources, the amount of required centralization of data can be greatly reduced by comparing the temporal locations of the CPs rather than comparing the entire signals [1]. This can be done blindly, without knowledge of the data contained in each CP [2]. However, since CPs occur at regular intervals, this leaves an integer ambiguity in each TDOA (any integer times the OFDM block length). In [1], the ambiguity was resolved by transmitting a small amount of data per block, whereas in this paper, we assume that the ambiguity can be resolved using integer least squares methods, as in [3].

Many papers discuss the CRLB of TDOA-based positioning [4]. Existing work assumes cross-correlation either between sensors [5], [6], [7] or with a known training signal [8]. In this paper, we consider the TDOA being estimated blindly and in a distributed fashion, by comparing the delays of the CPs (estimated as in [2]) in pairs of received signals. Thus, the main contribution of this paper is derivation of the CRLB on estimating the time delay of the CP within a symbol. Assuming that the integer ambiguity is appropriately resolved, this allows derivation of a CRLB on the TDOAs, and ultimately the CRLB for the underlying source localization problem, for the case in which no data is centralized aside from the reception times of the CPs.

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2. SYSTEM MODEL AND ML ESTIMATION

The model for the received signal is

$$y(t) = \sum_k x[k]p(t - \delta - kT) + n(t). \quad (1)$$

where δ is the time delay to be estimated, $x[k]$ is the transmitted data sequence, and $p(t)$ is the pulse shape with derivative $p'(t)$. We will use $p_o(t)$ to denote a time-normalized version of the pulse shape, i.e. $p(t) = p_o(t/T)$, and we assume that $p(t)$ is a raised-cosine pulse with excess bandwidth β .

The OFDM transmitter uses an FFT size of N and a CP length of ν samples. The transmitter sends one block of $M = N + \nu$ samples of $x[k]$ every MT seconds. The receiver uses an oversampling factor of q , and thus has a sampling period of $T_s = T/q$. The receiver samples a total of L blocks, or LqM samples. The set \mathcal{I}_c denotes the set of transmitted samples of $x[k]$ in cyclic prefixes (the first ν of each M samples), the set \mathcal{I}_d denotes the unique $N - \nu$ samples in the middle of each M samples, and the set \mathcal{I}_e denotes the last ν of each M samples (which get copied into the CP).

The signal power and noise power are σ_x^2 and σ_n^2 per sample, respectively, and the Signal to Noise Ratio (SNR) is σ_x^2/σ_n^2 . The transmitted data is assumed to be white (aside from the presence of the CP), and the noise is assumed to be completely white over the spectrum of interest.

In [2], a Maximum Likelihood (ML) algorithm was derived to jointly estimate the temporal location of the CP and the Carrier Frequency Offset (CFO), but no CRLB was derived. In this paper, for simplicity, we assume there is no CFO. The ML algorithm of [2], generalized¹ to allow for oversampling by q and averaging over L blocks, is given by

$$\hat{\delta}_{ML} = \arg \max_{-\frac{Mq}{2} \leq \delta < \frac{Mq}{2}} \left\{ \gamma(\delta) - \frac{\rho}{2} \Phi(\delta) \right\}, \quad (2)$$

$$\gamma(\delta) = \sum_{l=1}^L \sum_{k=Mql+\delta}^{Mql+\delta+\nu q-1} y(kT_s) y^*((k+Nq)T_s), \quad (3)$$

$$\Phi(\delta) = \sum_{l=1}^L \sum_{k=Mql+\delta}^{Mql+\delta+\nu q-1} (|y(kT_s)|^2 + |y((k+Nq)T_s)|^2), \quad (4)$$

$$\rho = \frac{SNR}{SNR + 1}. \quad (5)$$

¹The ML algorithm of [2] assumed $q = 1$, and it is not immediately obvious whether the generalization here is still ML when $q > 1$.

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In the next section, we will also consider an approximate ML algorithm, given by

$$\hat{\delta}_{apx} = \arg \max_{-\frac{Mp}{2} \leq \delta < \frac{Mp}{2}} \{\gamma(\delta)\}. \quad (6)$$

This is motivated by the fact that $\Phi(\delta)$ is approximately constant over δ , especially if the amount of averaging (L) is large. The $\rho \Phi(\delta)$ term also vanishes at low SNR.

3. EFFECTS OF SAMPLING

Intuitively, the CRLB should depend on the signal bandwidth $1/T$, but not on the actual sampling rate $1/T_s$, since the latter in part defines the estimation method. However, the performance of a given estimator will depend on the oversampling factor. For example, the ML algorithm of [2] does not interpolate between samples, hence its variance is at least that of a uniform random variable distributed over $[-T_s/2, T_s/2]$, which is $T_s^2/12$. This “sample resolution” bounds the Root Mean Squared Error (RMSE), causing a departure from the CRLB at high SNR values.

In order for the CRLB to be analytically tractable in a short paper, the derivation in the next section assumes sampling at the Nyquist rate, i.e. $q = 1$, which makes adjacent samples uncorrelated. Prior work on time delay estimation [5], [6] avoids this assumption by working in the frequency domain. However, it does not make sense to discuss the power spectra of OFDM systems, since the CP makes OFDM non-stationary (in fact, cyclo-stationary with period MT). However, similar work on image registration [9] makes an assumption analogous to ours, i.e. that the observed images are sampled at the Nyquist rate or greater.

4. CRLB DERIVATION

First, we consider the general case, then we derive the bound on estimators that only use the autocorrelation function of (3) to locate the CP. The latter derivation is cleaner but only applies to autocorrelation based estimators, e.g. (6). Real data is assumed for simplicity, and the SNR is assumed known.

4.1. General CRLB

In the general approach, the unknowns are the time delay δ and the LN nuisance parameters $\{x[k]\}$. Let \mathbf{x} consist of L blocks of N samples of $x[k]$ each, containing no repetition, i.e. omitting the CP samples. The log-likelihood of the received vector \mathbf{y} is

$$\mathbf{y} = [y(T), \dots, y(LMT)]^T \quad (7)$$

$$\underbrace{\ln(f(\mathbf{y}|\mathbf{x}, \delta))}_{\mathcal{L}} = \text{const.} - \frac{1}{2\sigma_n^2} \sum_m [y(mT) - \mathbb{E}\{y(mT)\}]^2, \quad (8)$$

$$\mathbb{E}\{y(mT)\} = \sum_k x[k]p(mT - \delta - kT). \quad (9)$$

The Fisher Information Matrix (FIM) has a block structure,

$$\mathbf{J} = \begin{bmatrix} \mathbf{A} & \mathbf{B}^T \\ \mathbf{B} & \mathbf{C} \end{bmatrix}, \quad (10)$$

where \mathbf{A} is 1×1 , \mathbf{B} is $LN \times 1$, and \mathbf{C} is $LN \times LN$. In each dimension, the first element of \mathbf{J} corresponds to δ , and then each

successive set of N elements corresponds to a block of (unprefixed) independent data samples $x[k]$.

The (scalar) submatrix \mathbf{A} , evaluated at $\delta = 0$ for simplicity, is

$$-\mathbb{E}\left\{\frac{\partial^2 \mathcal{L}}{\partial \delta^2}\right\} = \frac{1}{\sigma_n^2} \sum_m \left(\frac{\partial \mathbb{E}\{y(mT)\}}{\partial \delta}\right)^2 \quad (11)$$

$$= \frac{1}{\sigma_n^2} \sum_m z^2[m], \quad (12)$$

$$z[m] \triangleq \sum_k x[k]p'((m-k)T). \quad (13)$$

Again for $\delta = 0$, \mathbf{B} and \mathbf{C} are given element-wise by

$$-\mathbb{E}\left\{\frac{\partial^2 \mathcal{L}}{\partial \delta \partial x[k_0]}\right\} = \frac{1}{\sigma_n^2} \sum_m \frac{\partial \mathbb{E}\{y(mT)\}}{\partial \delta} \frac{\partial \mathbb{E}\{y(mT)\}}{\partial x[k_0]} \quad (14)$$

$$= \frac{-1}{\sigma_n^2} \sum_m z[m] \frac{\partial \mathbb{E}\{y(mT_s)\}}{\partial x[k_0]} \quad (15)$$

$$-\mathbb{E}\left\{\frac{\partial^2 \mathcal{L}}{\partial x[k_0] \partial x[m_0]}\right\} = \frac{1}{\sigma_n^2} \sum_m \frac{\partial \mathbb{E}\{y(mT)\}}{\partial x[k_0]} \frac{\partial \mathbb{E}\{y(mT)\}}{\partial x[j_0]}, \quad (16)$$

Due to the repetition induced by the CP,

$$\frac{\partial \mathbb{E}\{y(mT)\}}{\partial x[k_0]} = \begin{cases} p((m-k_0)T), & k_0 \in \mathcal{I}_d \\ p((m-(k_0-N))T) + p((m-k_0)T), & k_0 \in \mathcal{I}_e \end{cases} \quad (17)$$

For a Nyquist pulse shape, (17) becomes

$$\frac{\partial \mathbb{E}\{y(mT)\}}{\partial x[k_0]} = \begin{cases} \delta_K(m, k_0), & k_0 \in \mathcal{I}_d \\ \delta_K(m, k_0 - N) + \delta_K(m, k_0), & k_0 \in \mathcal{I}_e \end{cases} \quad (18)$$

where $\delta_K(\cdot)$ is the Kronecker delta function. Thus,

$$\mathbf{A} = \frac{1}{\sigma_n^2} \sum_m z^2[m] \quad (19)$$

$$\mathbf{B}[k_0] = \frac{1}{\sigma_n^2} \cdot \begin{cases} -z[k_0], & k_0 \in \mathcal{I}_d \\ -z[k_0 - N] - z[k_0], & k_0 \in \mathcal{I}_e \end{cases} \quad (20)$$

$$\mathbf{C} = \frac{1}{\sigma_n^2} \begin{bmatrix} \mathbf{I}_{N-\nu} & \mathbf{0} \\ \mathbf{0} & 2\mathbf{I}_\nu \end{bmatrix} \otimes \mathbf{I}_L, \quad (21)$$

where \otimes is the Kronecker product. (Note that k_0 only indexes the last N of each $N + \nu$ samples of x .) Using Schur complements to perform the matrix inversion in block fashion, the top left element of the CRLB is given by

$$\text{VAR}[\hat{\delta}] \geq [\mathbf{A} - \mathbf{B}^T \mathbf{C}^{-1} \mathbf{B}]^{-1} \quad (22)$$

$$= \sigma_n^2 \left[\sum_m z^2[m] - \sum_{m \in \mathcal{I}_d} z^2[m] - \frac{1}{2} \sum_{m \in \mathcal{I}_e} (z[m] + z[m - N])^2 \right]^{-1} \quad (23)$$

In (23), there are three summations over the index m . The first summation includes all m , including the CP set, \mathcal{I}_e ; the data set, \mathcal{I}_d ; and the data in the ends of blocks, \mathcal{I}_e . The second summation only includes the data in the middle of each block, and the third summation only includes the data in the ends of blocks.

Noting that “ $m \in \mathcal{I}_e$ ” is equivalent to “ $(m - N) \in \mathcal{I}_c$ ”,

$$\begin{aligned} \text{VAR}[\hat{\delta}] &\geq 2\sigma_n^2 \left[\sum_{m \in \mathcal{I}_e} (z[m] - z[m - N])^2 \right]^{-1} \\ &= 2\sigma_n^2 T^2 \left[\sum_{m \in \mathcal{I}_e} \left(\sum_k (x[k] - x[k - N]) p'_o(m - k) \right)^2 \right]^{-1}. \end{aligned} \quad (24)$$

Observe that the terms when $k \in \mathcal{I}_e$ are zero. Moreover, we always have $m \in \mathcal{I}_e$, and the pulse shape factor is nearly zero except when $|m - k|$ is small. Thus, the terms that contribute most to the summation are the boundary terms, i.e. when k is just outside of \mathcal{I}_e .

Equation (25) is the CRLB, and cannot be simplified further without approximations. However, in order to gain intuition, consider the case of large L , which enables the approximation

$$\sum_{m \in \mathcal{I}_e} (\cdot) = L \cdot \frac{1}{L} \sum_{l=1}^L \sum_{m \in \mathcal{I}_e^l} (\cdot) \approx L \cdot \mathbb{E} \left\{ \sum_{m \in \mathcal{I}_e^1} (\cdot) \right\}, \quad (26)$$

where the set \mathcal{I}_e^l is the data in block l of \mathcal{I}_e . Then, given that the data $x[k]$ is uncorrelated (aside from the repetition in the CP),

$$\begin{aligned} \text{VAR}[\hat{\delta}] &\gtrsim 2\sigma_n^2 T^2 \left[L \sum_{m \in \mathcal{I}_e^1} \sum_{k \notin \mathcal{I}_e} (2\sigma_x^2) (p'_o(m - k))^2 \right]^{-1} \\ &= \frac{T^2}{L \text{SNR}} \left[\sum_{m \in \mathcal{I}_e^1} \sum_{k \notin \mathcal{I}_e} (p'_o(m - k))^2 \right]^{-1} \\ &= \frac{T^2}{L \text{SNR}} \underbrace{\left[2 \sum_{d=1}^{\infty} d (p'_o(d))^2 \right]^{-1}}_{\eta}. \end{aligned} \quad (27)$$

For raised cosine pulse shapes with excess bandwidths β of 0, 0.25, and 0.5, the factor η is 0.32, 0.41, and 0.74, respectively.

4.2. Bound for autocorrelation-based methods

This section derives the CRLB for estimators that only use the autocorrelation data $\gamma(t)$, as in (6). Since $\gamma(t)$ are the observations, the only unknown is the time delay δ . The expected value of $\gamma(t)$ is

$$\Gamma(t) \triangleq \mathbb{E} \{ \gamma(t) \} = L \sigma_x^2 \Lambda(t - \delta), \quad (28)$$

where $\Lambda(t)$ is a triangular pulse spanning $-\nu \leq t \leq \nu$ with a peak height of ν . It can be shown that the variance of $\gamma(t)$ is

$$\text{VAR} \{ \gamma(t) \} = L\nu (\sigma_x^2 + \sigma_n^2)^2, \quad (29)$$

and that the auto-covariance of $\gamma(t_1)$ and $\gamma(t_2)$ is zero for all pairs of non-equal time instants.

Let $\boldsymbol{\gamma} = [\gamma(-M/2), \dots, \gamma(M/2 - 1)]$. The log-likelihood function of this vector given the delay δ is

$$\underbrace{\ln(f(\boldsymbol{\gamma}|\delta))}_{\mathcal{L}} = \text{const.} - \frac{\sum_t [\gamma(t) - \Gamma(t)]^2}{2L\nu (\sigma_x^2 + \sigma_n^2)^2} \quad (30)$$

The Fisher information is given by

$$J = -\mathbb{E} \left\{ \frac{\partial^2 \mathcal{L}}{\partial \delta^2} \right\} = \frac{\sum_t [\Gamma'(t)]^2}{L\nu (\sigma_x^2 + \sigma_n^2)^2}. \quad (31)$$

Given the triangular shape of $\Gamma(t)$, its derivative has a *magnitude* of $L\sigma_x^2/T$ for a range of 2ν samples, and is zero otherwise. Thus,

$$J = \frac{(2\nu) (L\sigma_x^2/T)^2}{L\nu (\sigma_x^2 + \sigma_n^2)^2}. \quad (32)$$

Inverting and simplifying,

$$\text{VAR}[\hat{\delta}_{\text{corr}}] \geq \frac{(1 + \text{SNR}^{-1})^2 T^2}{2L}. \quad (33)$$

The limiting cases are

$$\text{VAR}[\hat{\delta}_{\text{corr}}] \gtrsim \begin{cases} \frac{T^2}{2L}, & \text{high SNR} \\ \frac{T^2}{2L \text{SNR}^2}, & \text{low SNR} \end{cases}. \quad (34)$$

Interestingly (and counter-intuitively), both (27) and (33) are independent of the fraction of each block consisting of the CP, i.e. the fraction ν/M . Thus, a short CP is as good as a long CP for purposes of blind delay estimation in the absence of multipath.

4.3. CRLB on source localization

Assume K synchronized TDOA receivers work together to locate one OFDM source. Given the estimates of the CP locations and a suitable method for resolving the integer ambiguities [3], the TDOA between each pair of receivers can be obtained by subtracting the two estimated delays. This doubles the variance and the CRLB, and induces some covariance between TDOA estimates. Thus, the covariance matrix of efficient TDOA estimates and its inverse are

$$\mathbf{Q} = \frac{\eta T^2}{L \text{SNR}} [\mathbf{I} + \mathbf{1}\mathbf{1}^T] \quad (35)$$

$$\mathbf{Q}^{-1} = \left(\frac{\eta T^2}{L \text{SNR}} \right)^{-1} \left[\mathbf{I} - \frac{1}{K} \mathbf{1}\mathbf{1}^T \right] \quad (36)$$

where $\mathbf{1}$ is a $(K - 1) \times 1$ vector of ones.

From [7], the CRLB on the position estimate can be computed given the covariance matrix of a TDOA estimator, as

$$\text{CRLB}(x_{tx}, y_{tx}) = c^2 \left(\mathbf{G}^T \mathbf{Q}^{-1} \mathbf{G} \right)^{-1} \quad (37)$$

$$= \frac{\eta c^2 T^2}{L \text{SNR}} \left(\mathbf{G}^T \left[\mathbf{I} - \frac{1}{K} \mathbf{1}\mathbf{1}^T \right] \mathbf{G} \right)^{-1} \quad (38)$$

where \mathbf{G} is a $(K - 1) \times 2$ matrix that depends entirely on the geometries of the transmitter and receivers, with the i^{th} row (for $i = 2, 3, \dots, K$) given by

$$\mathbf{G}(\text{row } i) = \left[\frac{x_1 - x_{tx}}{r_1} - \frac{x_i - x_{tx}}{r_i}, \frac{y_1 - y_{tx}}{r_1} - \frac{y_i - y_{tx}}{r_i} \right] \quad (39)$$

where (x_{tx}, y_{tx}) and (x_i, y_i) are the locations of the source and K receivers, and r_i is the distance from the source to receiver i .

5. SIMULATIONS

The simulation parameters in this section are comparable to an IEEE 802.11a system: the FFT size is $N = 64$, the CP length is $\nu = 16$, the block size is $M = N + \nu = 80$, and the Nyquist sampling period is $T = 50$ ns. The number of blocks is $L = 10$ (yielding a 40 μ s observation time). The raised cosine pulse shape uses $\beta = 0$.

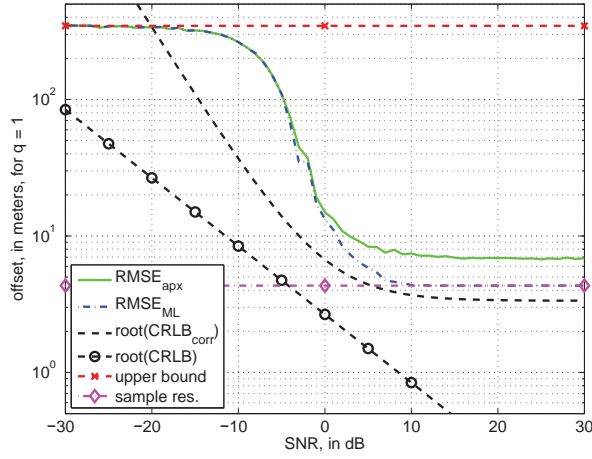


Fig. 1. Comparison of the CRLBs of (27) and (33) to the RMSEs of (2) and (6). The upper bound corresponds to a uniform distribution across a span of MT seconds, and “sample res” is the floor discussed in Section 3. There is no oversampling ($q = 1$).

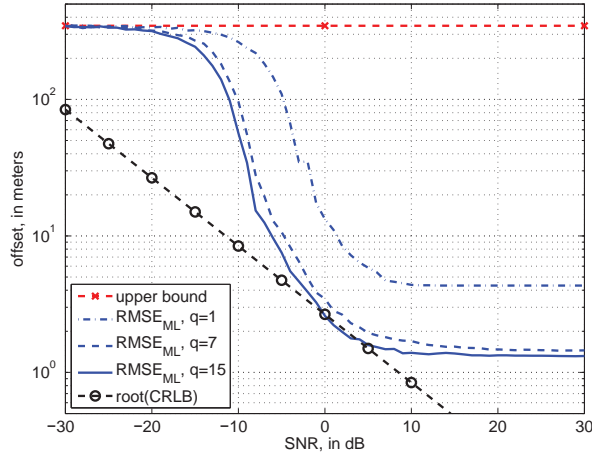


Fig. 2. Comparison of the CRLB of (27) to the RMSE of (2) for various oversampling factors q . The CRLB was derived assuming $q = 1$, but appears to hold for all q .

Fig. 1 shows the square roots of the CRLBs from (27) and (33), and the RMSEs of (2) and (6), using 1500 trials. The RMSE floor is as discussed in Section 3. Fig. 2 compares the performance of the ML estimator for various values of q to the CRLB. Although the CRLB was derived for $q = 1$, it appears to hold for all q . Sampling faster initially improves the performance of the ML estimator, but the realm of diminishing returns is reached very quickly. Fig. 3 and Table 1 compare the final RMSE and CRLB on position estimation, for $q = 7$. Ambiguity resolution was performed by dividing the space into $3 \text{ km} \times 3 \text{ km}$ regions and bounding the integer offsets within each region based on the sensor geometry and region size. The best choice of integers for each region was determined using box-constrained integer least squares [3]. The overall optimum integers were determined by selecting each region’s choice in turn, estimating the position via [10], and comparing the TDOAs that would result from that estimate to the observations.

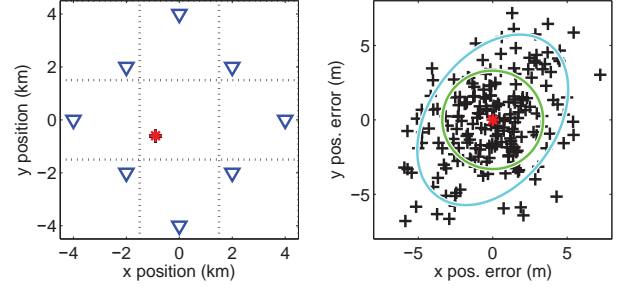


Fig. 3. Performance of position estimation, as in Section 4.3, for 0 dB SNR and $q = 7$. Each ∇ is a sensor, the $*$ is the source, and each $+$ is a resolved source estimate (200 trials). The outer and inner ellipses indicate the RMSE and the CRLB, both scaled up by 2 so that the former gives an 86% confidence interval.

Table 1. Effects of varying SNR on position estimation (in meters).

SNR:	-10 dB	-5 dB	0 dB	5 dB	10 dB	30 dB
RMSE	5790	11.5	3.82	2.26	1.85	1.47
$\sqrt{\text{CRLB}}$	50.7	5.07	1.67	1.01	0.81	0.66

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